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ABSTRACT

Research indicates that Hispanic technical students exhibit patterns of performance in problem solving tasks that are clearly distinct from those exhibited by Anglo students. This may be because problem solvers working problems in a nonnative language are in more danger of misinterpreting problems due to sentence construction, jargon, etc., than native speakers. These conclusions come from studying the performance of a group of 60 bilingual Hispanic students and a control group of 73 nonminority Anglo students, most of whom were engineering majors. Often, Hispanic students misinterpreted problems, even when familiar with all the vocabulary words used in the problem. In these cases, students necessarily translated a problem into mathematical terminology incorrectly, but in a manner totally consistent with the misinterpretation. Furthermore, Hispanic students were at a disadvantage in solving problems with speed, as measured by the number of questions not reached due to time constraints. These findings suggest that Hispanics would benefit most from problem solving courses that emphasize speed and accuracy in translating a word problem into mathematical notation. (KH)

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Teaching Problem Solving Strategies to Bilingual Students: What Do Research Results Tell Us?*

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Teaching Problem Solving Strategies to Bilingual Students: What Do Research Results Tell Us?

There has been a great deal of attention devoted to problem solving. Some researchers have attempted to analyze the problem solving process in an attempt to identify the various steps involved, while others have concentrated on the solver, in the hope of cataloguing successful as well as unsuccessful techniques used by both experts and novices. Despite all of the interest in problem solving, however, there is one area which I feel has received alarmingly little attention--namely, problem solving research as it pertains to minority populations. This is unfortunate in view of the underrepresentation of minorities in the technical professions.

For the past three years we have been studying the problem solving strategies employed by Hispanic bilingual college students in order to identify unique needs that this group may have when majoring in a technical field. Our investigations have naturally focused on problem solving, as this is an indispensable skill for anyone pursuing or practicing a technical profession. Thus far, results appear to indicate that Hispanic technical students exhibit patterns of performance in problem solving tasks that are clearly distinct from those exhibited by Anglo students. In this paper I will discuss the need for addressing several aspects of problem solving which are often neglected but which could be of great benefit to a bilingual population.

All of the results reported in this paper come from studying the performance of a group of 60 bilingual Hispanic students and, for purposes of comparison, a group of 73 nonminority Anglo students. The majority of the students in both groups were majoring in engineering, and although it could be argued that the two groups were not well matched in

several important academic and social characteristics, such as language ability and family income levels, that is not an issue for our purposes. Our hope is that by sharing our results with a pedagogic community, instructors may recognize the need to develop innovative methodologies for the instruction of problem solving techniques to Hispanic students.

Language and Problem Solving

No one would disagree that problem solving is a complex subject, and with the added dimension of bilingualism it is perhaps wise to examine some aspects of the process itself before discussing the individual solver. Let us then consider the four steps involved in solving a problem:

1. Understanding the problem,
2. Translating the problem into the appropriate mathematical terminology,
3. Working out the solution,
4. Checking the answer.

With slight variation these are the four steps that most would agree take place during problem solving. What interests me, perhaps more than most researchers, is something that is often downplayed in these four steps, and that is the role of language in the problem solving process. Whether a problem is stated orally or in written form, Steps 1 and 2 demand that the student process linguistic information precisely, since a failure to do so will result in an error before reaching Step 3. That a bilingual student will be as adept as a native speaker of the language in successfully executing Steps 1 and 2 may not be a valid assumption.

Problem solvers working problems in their nonnative language are in more danger of misinterpreting problems due to subtleties in sentence construction, jargon, etc. than native speakers of the language. What I have found in working with Hispanic bilinguals is that problems are often misinterpreted even when students are familiar with all the vocabulary words used in the problem. In these cases, a student will necessarily translate a problem into mathematical terminology incorrectly, but totally consistent with the misinterpretation.

I think a few examples will illustrate the situation. Consider the following problem:

> "In an engineering conference, nine meeting rooms each had 28 participants, and there were seven participants standing in the halls drinking coffee. How many participants were at the conference?"

This problem resulted in an error rate of 47% for the Hispanic group and 27% for the nonminority group. Among the errors, one particular answer common to both groups revealed two interesting misinterpretations. This answer consisted of multiplying nine rooms times the 28 participants, and subtracting the seven coffee drinkers, thereby obtaining an answer of 245 for the number of participants at the conference. The first misinterpretation consisted of taking the word "participants" to mean "those physically present at the meeting rooms listening to presentations", as opposed to the intended meaning of "all those who registered at the conference, no matter where they happen to be at the time of the presentations". The second misinterpretation consisted of assuming that the seven coffee drinkers came out of the nine meeting rooms. Thus, with these two misinterpretations the logical answer would be 245. The Hispanic group was much more prone to interpreting the problem in this fashion; 50% of all

Hispanic group errors were of this type, compared to only 26% of all nonminority group errors. Most of the errors committed by the nonminority group were arithmetic in nature, with 47% of all nonminority students who erred making a mistake in computing (9 x 28) + 7, compared to only 11% for the Hispanic group.

Another problem which revealed errors that derived from misunderstanding what was being asked is the following:

> "A carpenter bought an equal number of nails and screws for $5.70. If each nail costs $.02 and each screw costs $.03, how many nails and how many screws did he buy?"

The error rate on this problem for the Hispanic and nonminority groups was 57% and 42%, respectively. There was one type of error made almost exclusively by the Hispanic group. This error consisted of interpreting the first sentence of the problem to mean that an equal amount of money was spent for nails and screws out of the total $5.70 spent. Among all the errors committed by the Hispanic group, 35% were of this type, compared to only 1% of all nonminority errors. Other types of errors, such as making an arithmetic mistake after setting up the problem properly, or of obtaining the correct answer of 114 but thinking this was the combined number of nails and screws, halving this answer to obtain 57 nails and 57 screws, was made by approximately equivalent percentages of both groups.

Other language-related errors were found in a study where we specifically investigated the second step in the problem solving process, namely, that of translating a problem into the appropriate mathematical terminology (Mestre, Gerace, and Lochhead, 1982). Here, Hispanic technical students were asked to translate English and Spanish sentences that described a relationship between two variables into a mathematical equation.

At the time we conducted this study, we were already aware that certain relatively simple problems caused the now-famous "variable-reversal error". For example, in the problem,

> "Write an equation using the variables S and P to represent the following statement: 'There are six times as many students as professors at this university'. Use S for the number of students and P for the number of professors."

the error where a student would write the variable-reversed equation, $6S = P$, was committed consistently by about 30 to 40 percent of nonminority technical undergraduates (Mestre, et al., 1982; Clement, Lochhead, and Monk, 1981; Clement, 1982). The source of this kind of error derives from confusing labels and variables and has been well elucidated in a recent article by Rosnick (1981) in the Mathematics Teacher. Thus we expected, and found, the same kind of error for the Hispanic group. However, this group committed the variable reversal error with twice the frequency of the nonminority group.

Further, we discovered that certain errors made by the Hispanic group were virtually nonexistent for the nonminority group. In videotaped clinical interviews we were able to ascertain that some of these errors were linguistic in nature. For example, in the students and professors question, some students wrote $6S = 6P$. These students explained that the phrase "as many students as professors" meant an equal number of each; that is, $S = P$. The "6 times" in front of the statement was interpreted to mean that each side of $S = P$ should be multiplied by 6.

Other students wrote the equation $6S + P = T$. These students explained that this equation related the number of students, professors, and the total student-professor population, T, in the appropriate proportions. The fact that these students could be prompted to write the variable reversed equation, $6S = P$, after pointing out to them that the question asked for

a relationship between S and P showed that these students were having trouble ascertaining what the problem was asking.

Speed in Problem Solving

If the role of language in the problem solving process is often downplayed, then by comparison it is safe to say that speed in the problem solving process is something almost completely neglected. This is understandable. After all, most of us in our roles as instructors are much more interested in students being able to solve problems correctly, and not so much that they be able to do so in record speed. However, being able to solve problems quickly is a skill that would particularly benefit Hispanics, as the following results will demonstrate.

It is somewhat ironic that, having just finished the discussion of the role of language in problem solving in the last section, I will again emphasize language skills before launching into the discussion of speed in problem solving. Nonetheless, I think it would prove informative to first discuss a standardized language exam - the Test of Reading, Level 5, (Guidance Testing Associates, 1962). This exam contains three sections covering vocabulary, speed of comprehension, and level of comprehension, and it is not surprising that the Hispanic group scored below the nonminority group in all three sections (Mestre, 1981). The largest disparity in performance occurred in the speed of comprehension section. Here, students were required to read a sentence from which a word had been deleted and then select from among five choices that word which best fits in the space of the omitted word. This section is designed to measure the speed and accuracy with which a student can read and understand a sentence. The large disparity in performance between the two groups in this section was

primarily due to the ability of the nonminority group to answer significantly more questions in the allotted time than the Hispanic group. In the six minutes allowed for this section, the average number of questions attempted per student was 20.5 for the nonminority group, compared to 15.3 for the Hispanic group.

With this result in mind, let us now turn to mathematics performance. More specifically, I will discuss how students performed in two exams, the Short Algebra Inventory and the Word Problem Inventory. Covering topics such as solving simultaneous equations and factoring quadratic polynomials, the Short Algebra Inventory required little semantic processing, and was used to test a student's facility with algebraic manipulations. The Word Problem Inventory contained word problems requiring only a basic knowledge of algebra, but required substantial amounts of linguistic processing. A more detailed description of these two exams can be found in Mestre (1981).

Both groups' performance in these exams is summarized in Figure 1. Each bar graph area is representative of the total number of questions for a particular exam. The five categories are: 1) questions not reached due to time limitations, 2) totally correct responses, 3) errors deriving from not setting up the problem properly, 4) arithmetic errors after the problem was set up properly, and 5) answers not categorizable into the previous four categories.

Figure 1 emphasizes the two largest areas of difficulty for the Hispanic group. First, we can see that the Hispanic group is at a disadvantage in problem solving speed, as measured by the number of questions not reached due to time constraints. The second area in need of improvement for this group is in the number of incorrect answers due to improperly setting up a problem. In the case of the Word Problem Inventory, this latter category contains the errors deriving from language misinterpretations.

Discussion and Conclusions

Before making a few suggestions which may prove effective for improving the problem solving skills of Hispanic technical college students, I would like to recapitulate what our research findings tell us about the special needs of these students, and discuss how traditional methods for teaching problem solving fail to address these needs. Our results indicate that language plays a crucial role in problem solving for Hispanics, especially for word problems requiring large amounts of linguistic processing. More specifically, the probability of misinterpreting problems is higher for these students than for nonminority Anglo students. Further, slower reading speed and comprehension contribute to the slower problem solving speed exhibited by Hispanics.

Next, let us analyze the content of typical problem solving courses. The typical content of problem solving courses consists of instructors working out numerous examples in the hope that students will emulate the techniques demonstrated and of giving students many problems to solve. Not only does this approach fail to emphasize the needs of Hispanics, but it also contains two flaws. First of all, the procedures presented by instructors in solving sample problems are often chosen due to clarity and elegance, and are not necessarily the procedures that the instructor originally used to obtain the solution. Students are not shown an expert's technique for approaching a problem, but rather a refined approach. Secondly, I will not be the first to point out that although working out a large number of problems is a necessary condition for problem solving proficiency, this does not necessarily mean that it is a sufficient condition as well (Kilpatrick, 1978).

It is my belief that Hispanics would benefit most from problem solving courses that emphasize speed and accuracy in translating a word problem into

mathematical notation. A complaint that has often been voiced by Hispanic students during my conversations with them is that they usually do not have adequate time to finish all of the problems given in the 50 minutes of the typical "hour exam", especially when they spend a considerable amount of this time attempting to understand what is being asked. Drill work in the first two of the four problem solving steps listed earlier, namely, in understanding the problem and in translating the problem into the appropriate mathematical terminology, would help alleviate this situation. Grappling with problems which demand substantial amounts of linguistic processing would train these students to glean the important information from a word problem quickly, and to appropriately "set up" the problem while allowing the instructor to identify and address possible misinterpretations.

Recognition that language plays a very important role in cognitive processes is not a recent discovery. During the first half of this century, psychological theories such as Vygotsky's and linguistic theories such as Whorf's have posited that intellectual functions may be heavily mediated by linguistic processes (for a good review, see Segalowitz, 1977). The fact that recent research results have found high correlations between problem solving and language skills (Krulik, 1980), particularly for bilingual populations (Duran, 1979; Mestre, 1981), has rekindled interest in the role of language in the problem solving process. Perhaps we have been separating language skills and mathematical skills when in fact we should be integrating them. The current situation is best described in a recent article in the Mathematics Teacher by Henrichs and Sisson (1980):

> "Mathematics teachers, along with teachers of other content areas have become increasingly sensitive to their students' deficiencies in reading and study skills. Before an attempt is made to remediate this problem, we must challenge the traditional practice of treating reading as a separate subject primarily taught at the elementary level. Reading must be integrated with course content".

Lest I leave the reader with the wrong impression, I would like to point out that even though I have been emphasizing the roles of language and speed in the problem solving process, this should not be taken to imply that other aspects of problem solving should be ignored. The performance of the Hispanic group in the Short Algebra Inventory as shown in Figure 1 indicates a need for improvement in algebraic skills for this group. The large number of errors in the "problems set up incorrectly" category cannot be attributed to language effects, given the small amount of linguistic processing required in this exam. Checking answers for reasonability also appears to be lacking. Students who wrote 6S = P in the students and professors problem may have questioned their answer had they substituted S = 1 to realize that in the hypothetical school with 1 student, there are 6 professors in contradiction of the problem statement. In the carpenter problem, those who assumed that $2.85 each was spent on nails and screws, and divided $2.85 by the price of one nail, $.02, to get 142½ for the number of nails bought, should have realized that hardware stores are not likely to sell anybody half a nail, no matter how eccentric the buyer is.

The road to becoming an "expert solver" is a long one. Not only is it necessary to have a mastery of the subject matter, but also involves elements of care, patience, persistence, imagination, experience, and sometimes a bit of luck. An improvement in any of these areas will bring the neophyte closer to the rank of expert.

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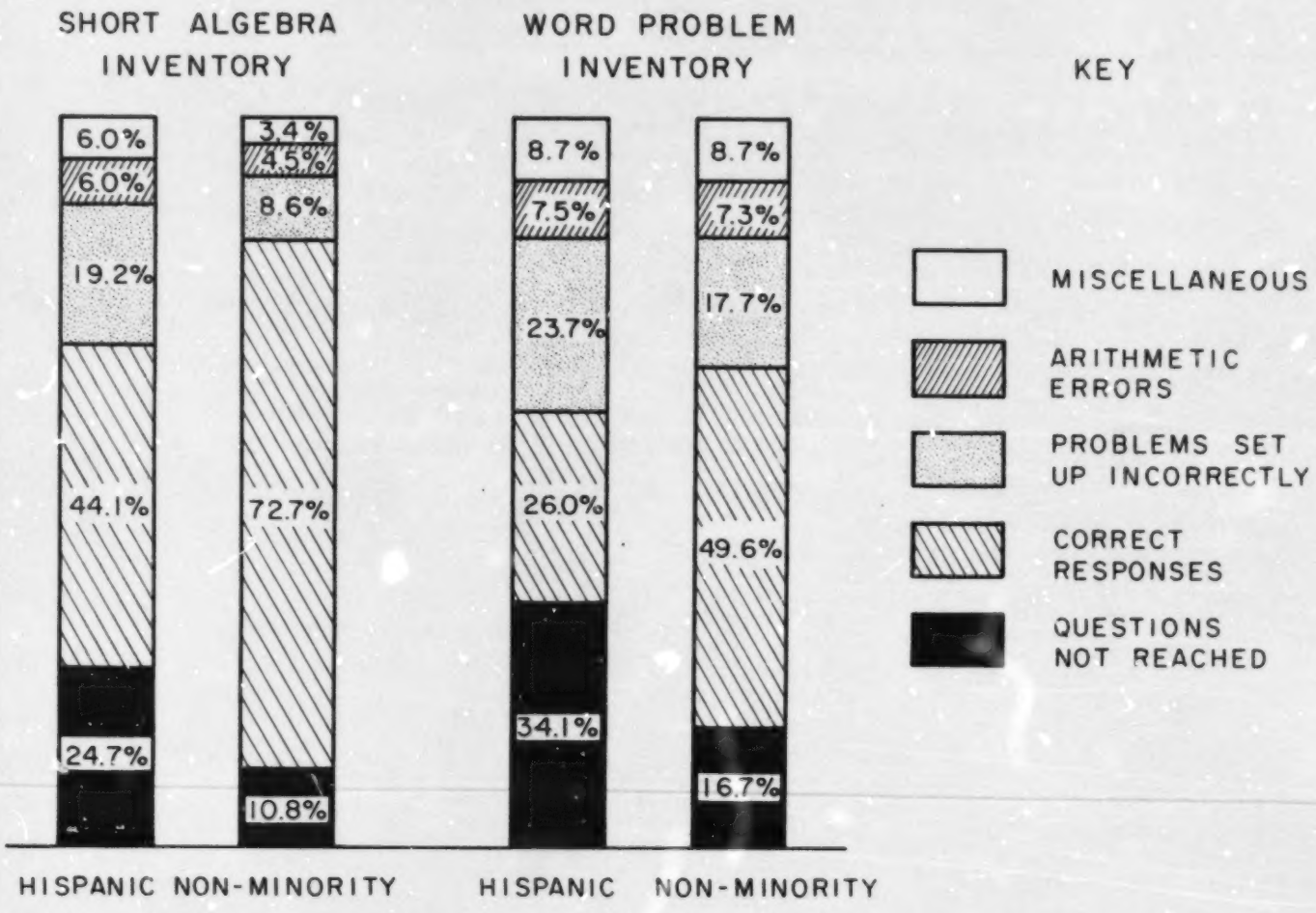
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MISCELLANEOUS
ARITHMETIC ERRORS
PROBLEMS SET UP INCORRECTLY
CORRECT RESPONSES
QUESTIONS NOT REACHED
HISPANIC
NON-MINORITY
HISPANIC
NON-MINORITY